

MATHEMATICS EXAMINATION FOR S5MPC /100Marks

Attempt all questions of section A (55Marks)

1. Use addition and subtraction formulae to find $\sin \frac{\pi}{12}$ /2Marks
2. Express $\cos 4x$ in function of $\sin x$ /4Marks
3. The table shows the value of function at a set of points

x	0.9	1	1.1	1.2
$f(x)$	0.266	0.242	0.218	0.198

Use linear interpolation to find: (a) the value of $f(1.04)$ /2Marks

(b) the value of x corresponding to $f(x) = 0.25$ /2Marks

4. If $\cos A = -\frac{7}{25}$, find the values $\tan \frac{A}{2}$ /3Marks
5. Prove that: $\sin 10^\circ \sin 50^\circ \sin 70^\circ = \frac{1}{8}$ /4Marks
6. Which of the following sequences converge, and which ones diverge? Find the limit of each convergent sequence.

(a) $\{8 - 2n\}_{n=1}^{+\infty}$ /1.5Marks

(b) $\{2 + (0.1)^n\}$ /1.5Marks

7. Solve the following equations: $\begin{cases} 2 \log_x y = 1 \\ xy = 64 \end{cases}$ /3Marks

8. Solve $2 \cos^2 x - 5 \sin x + 1 = 0$ for $x \in [0, \pi]$ /3Marks

9. Suppose your parents invest 10000frw in a saving account for a college at the time you are born. The average interest rate is 4% and is compounded quarterly. How much money will be in the college account when you are 18 years old? /3Marks

10. The sum of 3 consecutive terms in arithmetic progression is 33 and their product is 1287. What are those numbers? /3Marks

11. A child building a tower with blocks uses 15 for the bottom row. Each row has 2 fewer blocks than the previous row. Suppose that there are 8 rows in the tower.

- a) How many blocks are used for the top row? /1Mark
- b) What is the total number of blocks in the tower? /2Marks

12. Solve the following logarithmic equation: a) $3 \sin x + \sqrt{3} \cos x = 3$ /3Marks

b) $\log(x+2) + \log(x-1) = 1$ /3Marks

13. The radioactive element carbon-14 has a half-life of 5,750 years. If 100 grams of this element are present initially, how much will be left after 1000 years? /4Marks

14. The amount, $A(t)$ grams, of radioactive material in a sample after t years is given by

$$A(t) = 80 \left(2^{-\frac{t}{100}} \right).$$

- Find the amount of material in the original sample. /1Mark
- Calculate the half-life of the material (the half-life is the time taken for half of the original material to decay). /2Mark
- Calculate the time taken for the material to decay 1gram. /2Marks

15. Light travels from air into an optical fiber with an index of refraction of 1.44

- In which direction does the light bend? /1Mark
- If the angle of incidence on the end of the fiber is 22° , what is the angle of refraction inside the fiber? /2Marks
- Sketch the path of light as it changes media. /2Marks

Attempt any three questions of section B (45Marks)

16. Consider the sequence $\{u_n\}$ defined by $u_0 = 0$ and $u_{n+1} = u_n + \frac{1}{2^n}$. Consider another sequence $\{v_n\}$ defined by $v_n = u_{n+1} - u_n$ Sequences

- Show $\{v_n\}$ is a geometric sequence and find its common ratio and first four terms. /8M
- Express $\{v_n\}$ in terms of n and the sum of 20 first terms of sequence $\{v_n\}$ /7Marks

17. $\sin x + \sin 3x < -\sin 2x$ /15Marks

18. Prove that: $\frac{\sin 5x - 2\sin 3x + \sin x}{\cos 5x - \cos x} = \tan x$ /15Marks

19. Prove that $\sqrt{2 + \sqrt{2 + \sqrt{2 + 2\cos 8x}}} = 2\cos x$ /15Marks

20. Consider the equation $x^2 - 3x + 1 = 0$

(a) Show that $x^2 - 3x + 1 = 0$ has one root lying between 0 and 1 and another lying between 2 and 3. /3Marks

(b) Show that $x^2 - 3x + 1 = 0$ can be rearranged into the form: $x = \frac{x^2 + p}{q}$

where p and q are constants and $x = r + \frac{s}{x}$ where r and s are constants. State the values of p , q , r and s . /6Marks

(c) Using the iteration formula $x_{n+1} = \frac{x_n^2 + p}{q}$ together with your values of p and q . Starting at $x_1 = 0.5$, find one root of $x^2 - 3x + 1 = 0$ for 3 decimal places. 3Marks

(d) Using the iteration formula $x_{n+1} = r + \frac{s}{x_n}$ together with your values of r and s . Find

the second root of $x^2 - 3x + 1 = 0$ for 3 decimal places. 3Marks

GOOD LUCK!!!!